

# Fundamentals of Astrodynamics and Applications 2<sup>nd</sup> Ed 2<sup>nd</sup> printing

## Errata

February 18, 2006

This listing is an on-going document of corrections encountered in the book. I appreciate any comments and questions you find. I use RHS for right hand side when referring to equations. You may reach me at: [dvallado@agi.com](mailto:dvallado@agi.com) or [dvallado@msn.com](mailto:dvallado@msn.com).

- Page iii**, front, Equations: The parabolic radius should be  $p/2$  instead of  $q$ . The Hyperbolic mean anomaly should be  $M = e\text{SINH}(H) - H$ , and not have “ $F$ ”.
- Page 39**, Eqs : Delete the “ $i$ ” summation in the first equation. Multiply by  $1/2$  before  $G$  in the second equation. Delete the “ $i$ ” summation in the third equation. Multiply by  $1/2$  before  $G$  in the last equation.
- Page 53**, Last para : Insert a divided by sign in “ $(14)(2\pi) / (86400)$ ”
- Page 64**, Next to last Eq : Insert parentheses around the first term on the first line,  $\text{SINH}(H)$ . Insert a single start paren before  $-e\text{SINH}(H)$  on the second line.
- Page 70**, Eq for  $n\Delta t$  : Insert  $c_3$  outside the first term parentheses in the first term on the rhs and  $c_2$  in the second term.
- Page 81**, Fig 2-9 : Change “ $e = 2.0$ ” to “ $e = \sqrt{2}$ ”.
- Page 85**, Next to last sentence : Change reference to Eq. (2-94) to Eq. (2-92).
- Page 93**, End of pg : Add “( $c_{int}$  is a constant of integration)”.
- Page 94**, Top 2 sets of equations : Change  $c_o$  to  $c_{int}$  in the first set of equations. Change  $\chi_o$  to  $\chi$  in Eq 2-67.
- Page 95**, Eq for  $E$  at the bottom of the page : Change the sign in the denominator to  $1 + e \cos(v)$ .
- Page 96**, Eq 2-69 : Delete  $1/2$  from the denominators and change  $\chi$  to  $\psi$  in the  $c_2$  equation.
- Page 109**, top of page : The references to “elliptical equatorial” orbits should say “non-circular” orbits.
- Page 111**, Sentence before Eq 2-91 : Change to read “pgs 53-54”.
- Page 113**, Ref after Eq 2-93 : Change reference to Eq C-9 to Eq. C-8.
- Page 132**, First sentence : Change to “Use Eq. (C-12), simplify, and rearrange to get” and “Obtaining common denominators and using Eq. (2-104) results in”.
- Page 145**, end of page : Insert the following - (note the approximate result because  $h_{ellp}$  is not measured directly along the geocentric radius).
- Page 146**, Eq 3-10 : Insert  $\cong$  in place of  $=$ .
- Page 150**, Eq 3-14 and Ex 3-2 : Delete  $R_{\oplus}$  in Eq 3-14 and in Ex 3-2. Change the sentence to “Using Eq. (3-7) and  $R_{\oplus} = 6378.137$  km, find the interim values for  $C_{\oplus} = 6378.541\ 012\ 74$  and  $S_{\oplus} = 6335.840\ 635\ 42$  km,  $h_{ellp} = 0.056$  km.”.
- Page 154**, Last line: Change the equation to be “ $LHA = GHA + \lambda$ ”.
- Page 155**, Caption for Fig 3-9 : Add “observer to the” object of interest.
- Page 168**, Sentence before middle equation : Insert “the example” after “The result for”.
- Page 183**, Reference before Eq 3-37 : Change “Sec 2.2” to “Sec 5.1.1”.

**Page 203, 205**, Ex problems : Change the “DayofYr)12” to “DayofYr)24”.

**Page 205**, Section 3.7 : This section is being re-written to include the information from Vallado, Seago and Seidelmann (2006) on the new IAU Resolutions.

**Page 210**, Eq 3-51 : Change the signs on the cubed terms to be

$$M_{\odot} = 357.529\ 109\ 18^{\circ} + (99r + 359.050\ 291\ 1)T_{TDB} - 0.000\ 153\ 7T_{TDB}^2 + 3.8 \times 10^{-8}T_{TDB}^3 \\ - 3.19 \times 10^{-9}T_{TDB}^4$$

$$u_{M\zeta} = 93.272\ 090\ 62^{\circ} + (1342r + 82.017\ 457\ 7)T_{TDB} - 0.003\ 542\ 0T_{TDB}^2 - 2.88 \times 10^{-7}T_{TDB}^3 \\ + 1.16 \times 10^{-9}T_{TDB}^4$$

**Page 216**, Middle Eq and Fig 3-25 : Insert “(the frame bias is the second line in the following equation)” before the middle equation. Delete the  $0.025\ 24T_{TT}$  term on the last line of the middle equation. Add the frame bias (B) before and after the GCRF in Fig 3-25.

**Page 251**, Eq 4-3 : Change the sign in the velocity equation to be  $\dot{v}_{ECI} = \dot{\rho}_{ECI} + \dot{v}_{SiteECI}$ .

**Page 254**, The first two paragraphs are redundant. and the following equation is missing from the end of the page. (See page at end of this listing)

IF Elevation  $\neq 90^{\circ}$

$$\sin(\beta) = \frac{\rho_E}{\sqrt{\rho_S^2 + \rho_E^2}} \quad \cos(\beta) = \frac{-\rho_S}{\sqrt{\rho_S^2 + \rho_E^2}}$$

**Page 262**, Ex 4-1, equation for rdot. : Change the solar distance to 149,597,870.

**Page 272**, Ex 5-2 : Add “Let  $T_{TDB} \equiv T_{UT1}$  and be sure to use  $\zeta = 90^{\circ}\ 50''$ ”.

**Page 299**, Fig 5-11 : Insert the units of Lux for illumination on the vertical axis and add a footnote saying “Luminous emittance (or Lux = lumen / m<sup>2</sup>) is a measure of the visible light emitted”.

**Page 320-321**, First sentence : Change the reference from Eq 2-73 to Eq 2-74.

**Page 355**, Alg 46 parameters : Change  $\lambda_{true\ t_{gt1}}$  to  $\lambda_{true0}$ .

**Page 405**, Middle para : Change *Montaliche Correpondenz* to *Monatliche Correspondenz*.

**Page 410**, First Eq : Change ECEF to ECI, add an additional “site” subscript to the  $r$  vector, and insert  $\equiv$  in place of =.

**Page 412**, Last Eq : Add an additional site subscript to the  $r$  vector in the velocity Eq.

**Page 421, 422, 428**, Eq for  $L^{-1}$ : Change the signs of the 1-2, 2-1, 2-3, and 3-2 terms (row-col).

**Page 441**, Given data for the Ex : Insert a + sign between the  $J$  and  $K$  components of  $r_3$ .

**Page 461**, Last Eq for  $g$  : Insert “-  $t_o$ ” after the  $t$ . Change  $\chi_o$  to  $\chi$  in each term.

**Page 462**, Eq 7-47 : Change  $\chi_o$  to  $\chi$ .

**Page 463**, Eqs : Insert “-  $t_o$ ” after the  $t$  in the second and third Eqs. The numerator should be “ $f\dot{g} - 1$ ” in the fifth Eq. Change  $\chi_o$  to  $\chi$  in the first three equations.

**Page 465**, Alg 55 : Change  $\Delta t_o$  to  $\Delta t$  on the first line. Change  $\chi_o$  to  $\chi_n$  in the loop. Change  $\Delta t$  to  $\Delta t_n$  and  $\Delta t_o$  to  $\Delta t$  for the remainder of the algorithm.

**Page 470**, Table 7-2 : Insert “n” subscripts for  $\chi$ ,  $\Delta t$  (TU), and  $\Delta t$  (min) in the table heading.

**Page 503**, Sentence before Fig 8-2 : Say ‘fourth-order “global” error’.

**Page 523**, Fig 8-6: The bold latitude lines should be spaced as in Fig 8-4, and the label should be “10:4” instead of “4:3”.

**Page 529**, Atmospheric density, Sec 8.7 and Section 8.8 : These sections are being re-written to include the information from Vallado (2005) about force models. An improved discussion of numerical techniques is presented, and the practical applications is re-written.

**Page 541**, Last sentence before the section on the Russian GOST model : Change “will be” to “is”.

**Page 543**, Eq 8-36: The multiplier on the final term should be “15/2” instead of “15”.

**Page 545**, Eqs : Insert a summation from  $m = 0$  to  $l$  after the summation in Eq 8-39. The first term in the brackets of Eq 8-40 should have a vector symbol. The first term of the next to the last equation should have a vector symbol in the denominator. There should be no vector symbol on the first term of the last equation in the numerator.

**Page 578**, Last equation : Move the dot above the final partial of  $x$ .

$$\frac{d^2\hat{x}(\hat{c}, t)}{dt^2} = \frac{\partial^2\hat{x}(\hat{c}, t)}{\partial t^2} + \sum_{i=1}^6 \frac{\partial\hat{x}(\hat{c}, t)}{\partial c_i} \frac{dc_i}{dt}$$

**Page 582-583**, Eqs : Insert vector symbols for the position and velocity vectors in the second eq on pg 582. The summation index on the 4th equation should be “j” not “i”. The indices on the final term of the 5th equation should be “i” and “k”, not “k” and “i”. The dots should be removed from the velocity terms in the last equation on pg 582, and the first equation on pg 583.

**Page 587**, First sentence : The derivative in the denominator should be with the second term, not the first.

**Page 589**, Next to last equation: The negative sign is aligned with the first term on the rhs.

**Page 605**, Last sentence : The reference should be to Eq C-12.

**Page 610**, First equation : The constant for the multiplier of the last term should be 9 instead of 2.

**Page 622**, First sentence : The inequality should have a script “l”, “ $(m \leq l)$ ”.

**Page 660**, Section 9.9 : Section 9.9 is being re-written to include information from Vallado (2005) as well as a better introduction to the types of force model errors and sensitivities.

**Page 687**, 2nd and 3rd equations : The quantities after the summation should both be squared in the 2nd equation. The first quantity should be squared in the 3rd equation, and the first argument in the last term of the third equation should be  $\xi_j$ .

**Page 691**, Sentence before eigenvalue expansion : Add “(because the matrix is symmetric ( $a_{ij} = a_{ji}$ ) several terms can be simplified)”.

**Page 704**, Last paragraph : The equation for  $y_n$  should have an additional “i” subscript.

**Page 706**, Bottom of the page. In the definition for the STM, the “state” should be “state Errors” in two places, with the appropriate symbols (lowercase dx).

**Page 718**, Sentence before final covariance equation. Delete “state update and” before “covariance.”

**Page 724-740**, Several places. “state update” should be “state errors”. This includes labels in algorithms 64 and 65.

**Page 727**, Sentence before first state error equation. It should say “state errors” instead of “state”.

**Page 728**, Paragraph in “Types of Filters”. Delete the sentence beginning “Filter algorithms ...”

**Page 735**, Fig 10-9 caption. Replace “accuracy” with “uncertainty” in the final sentence.

**Page 743**, Last para. Add a footnote indicating: “As we mentioned on Pg 90, there is a difference between a STM that moves the state ( $\Phi$ ) and the state errors ( $\Phi$ ). The F matrix in this section is exclusively for the latter.”

**Page 750**, Add a condensed section on the state propagation using the state transition matrix.

**Page 773**, Fig 11-5 : Change the ‘y-axis’ labels from “4000” to “5000”, “5000” to “10000”. (See page at end of this listing)

**Page 785**, Middle paragraphs : The reference to Eq 6-12 should be to Eq 6-11, and the next reference to Eq 6-11 should be to Eq 6-12.

**Page 789**, Before the section on Nodal drift. Add the following.

To complete the analysis of Sun synchronous orbits, we can also solve Eq. (11-14) for the other parameters. Thus, if we are given semimajor axis and eccentricity, or semimajor axis and inclination, we can solve for inclination, or eccentricity, respectively.

$$i = \cos^{-1} \left( \frac{a^{7/2}}{3R_{\oplus}^2 J_2 \sqrt{\mu}} \frac{2(1.606378006 \times 10^{-4})(1-e^2)^2}{1} \right) \quad (11-15)$$

$$e = \sqrt{1 - \sqrt{\frac{-3R_{\oplus}^2 J_2 \sqrt{\mu} \cos(i)}{2(1.606378006 \times 10^{-4})}} a^{7/2}} \quad (11-16)$$

**Page 796**, Main Paragraph : Change “+ 3Δ*ρ* = 5Δ*ρ*” to “+ 2Δ*ρ* = 4Δ*ρ*.” and “5Δ*ρ*” to “4Δ*ρ*” in the next sentence. Delete “+*j*” in the final equation.

**Page 797**, Table 11-3 : Subtract one period quantity (Δ*ρ*) from each entry in columns 3 and 4. (See page at end of this listing)

**Page 864**, Definition of  $\Phi$  . Insert “error” before State Transition Matrix and add  $\Phi_S$  for State Transition Matrix.

**Page 869**, Eq B-5. The first “plus” sign on the rhs should be a “minus”.

**Page 874**, Equation for  $T(100)$ . The two occurrences for ( $T_x - T_o$ ) should be deleted, and a “ $T_x$ ” should be added before the “-0.94585589” term.

**Page 875**, Eq B-13. The first equation should be changed to

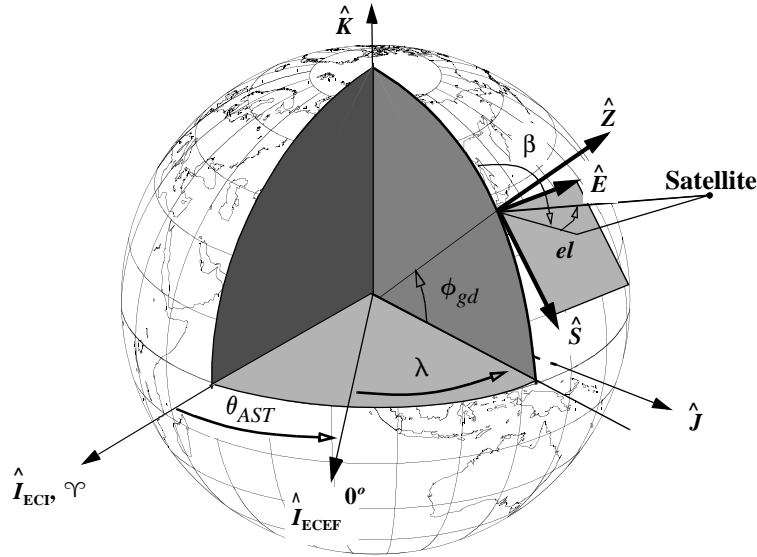
$$\rho_{std} (h_{ellp}) = \sum_{i=1}^5 \rho_i(125) \left( \frac{T_x}{T(h_{ellp})} \right)^{1+a_i+\gamma_i} \left( \frac{T_{corr}-T(h_{ellp})}{T_{corr}-T_x} \right)^{\gamma_i} .$$

**Page 894**, after the first sentence. Add “Note that because  $\sin(\phi)$  is always positive, the two quadrant solutions of cosine will also be positive.”

**Page 905-906**, Table D-1 and D-2: Switch the column heading to be “ $c_i = -J_i$ ” in Table D-1. Delete “unnormalized” from the heading in Table D-2 and insert “Both normalized and unnormalized values are shown” in the caption.

**Page 909**, Table D-5 : The  $J_3$  values for Jupiter and Saturn should be in the  $J_4$  row.

**Page 910**, Table D-6 : Switch the terms so “longitude terms ( $A_i, B_i$ ) and the obliquity terms ( $C_i, D_i$ )”.



**Figure 4-5. Topocentric-Horizon Coordinate System (SEZ).** We measure azimuth,  $\beta$ , and elevation,  $el$ , in the topocentric-horizon coordinate system. Most analyses consider the differences between ECI and ECEF coordinate systems.

$$\begin{aligned}\vec{\rho}_{SEZ} &= [\text{ROT2}(90^\circ - \phi_{gd})][\text{ROT3}(\lambda)]\vec{\rho}_{ECEF} \\ \dot{\vec{\rho}}_{SEZ} &= [\text{ROT2}(90^\circ - \phi_{gd})][\text{ROT3}(\lambda)]\dot{\vec{\rho}}_{ECEF}\end{aligned}$$

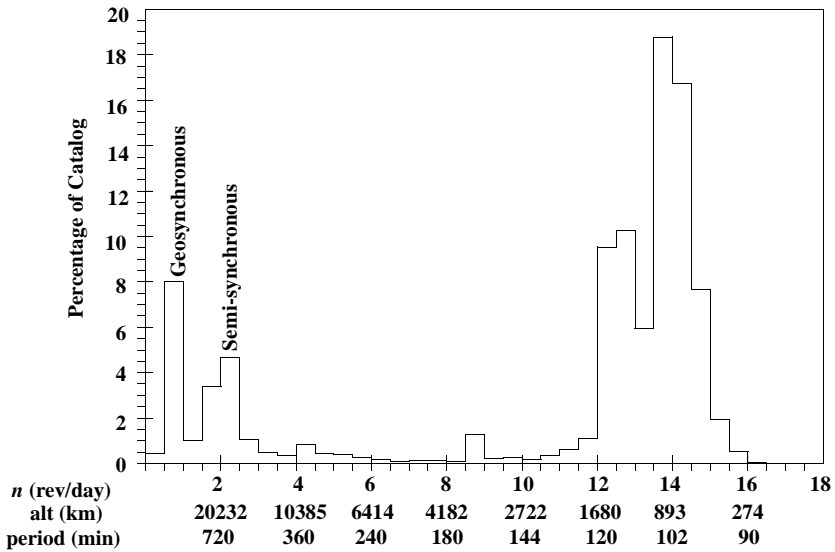
The next task is to find the individual observation quantities (range, azimuth and elevation). Once we know the slant-range vector, the magnitude is available. Determine the elevation from looking at Fig. 4-5. The sine function permits unique determination because elevation is limited to  $-90^\circ$  to  $90^\circ$ . Both expressions are given for completeness.

$$\sin(el) = \frac{\rho_Z}{\rho} \quad \cos(el) = \frac{\sqrt{\rho_S^2 + \rho_E^2}}{\rho}$$

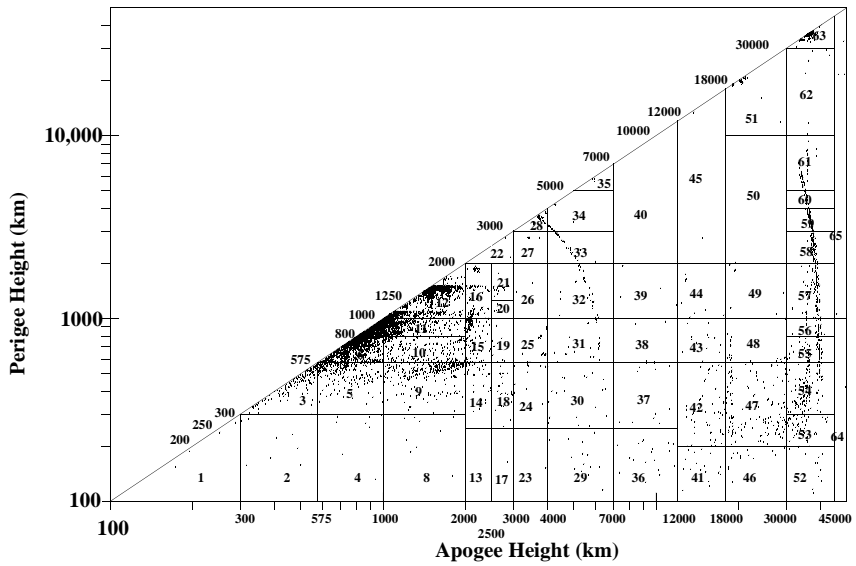
Due to the direction of positive measurement (positive clockwise from north), the relation for azimuth is somewhat different from the formulas for right ascension (Algorithm 25).

IF Elevation  $\neq 90^\circ$

$$\sin(\beta) = \frac{\rho_E}{\sqrt{\rho_S^2 + \rho_E^2}} \quad \cos(\beta) = \frac{-\rho_S}{\sqrt{\rho_S^2 + \rho_E^2}}$$



**Figure 11-4. Mean Motion of Earth-Orbiting Satellites.** We determine two important quantities from mean motion: the orbital period and semimajor axis. The semimajor axis minus the Earth’s radius gives us the altitude for a circular orbit for the given mean motion,  $n$ .



**Figure 11-5. Categorizing Satellite Orbits.** This figure shows the distribution of satellite orbits. The categories are numbered sequentially from the left (1), proceeding to the right, and then vertically up. Apogee and perigee altitudes values (km) are also shown.

**TABLE 11-3. Effect of Drag on a Satellite's Period.** The effect of drag on a satellite's orbital period is shown as a periodic effect (the change in nodal period on each revolution), a difference from the previous revolution, and the accumulated effect from the original period.

| Revo-<br>lution # | Nodal Period at<br>End of Revolution<br>$\rho_i$           | $\Delta t = \Delta t_{diff} + \Delta t_{rev}$<br>over 1 Revolution | Accumulated $\Delta t_{acc}$<br>from Revolution 0   |
|-------------------|--|--|---|
| 0                 | $\rho_o$   | 0  | 0   |
| 1                 | $\rho_1 = \rho_o - \Delta\rho$                             | $\Delta\rho/2$   | $\frac{\Delta\rho}{2}$  |
| 2                 | $\rho_2 = \rho_1 - \Delta\rho$<br>$= \rho_o - 2\Delta\rho$ | $\Delta\rho + \Delta\rho/2$  | $\frac{3\Delta\rho}{2} + \frac{\Delta\rho}{2} = 2\Delta\rho$                                    |
| 3                 | $\rho_3 = \rho_2 - \Delta\rho$<br>$= \rho_o - 3\Delta\rho$ | $2\Delta\rho + \Delta\rho/2$                                       | $\frac{9}{2}\Delta\rho$   |
| $j$               | $\rho_j = \rho_o - j\Delta\rho$                            | $(j-1)\Delta\rho + \Delta\rho/2$                                   | $\sum_{k=1}^j \left( (k-1)\Delta\rho + \frac{\Delta\rho}{2} \right) = \frac{1}{2}j^2\Delta\rho$ |

### Planning Orbit-Maintenance Maneuvers

Let's consider a strategy for developing an orbit-adjustment burn plan to maintain our repeat groundtrack. We'll consider perturbations in the elements  $a$  and  $i$  that affect the nodal period (drag) and the nodal regression drift rate (central-body and third-body). We won't consider the effects of perturbations in the eccentricity because they're usually small for repeat-groundtrack orbits of near-Earth satellites in nearly circular orbits.

As we have seen, perturbations acting on the satellite's orbital motion cause the equator crossing points to drift in longitude. The main cause of this groundtrack drift at the equator is decay in the semimajor axis caused by atmospheric drag, as well as drift in the orbital inclination due to the effect of the Sun's gravitational attraction. Assuming small perturbations in the spacecraft's motion so we can neglect higher-order terms, the groundtrack drift after one revolution is given by a truncated Taylor series expansion [using Eq. (11-23)]

$$\begin{aligned} \Delta\lambda_s &= \frac{\partial\lambda_s}{\partial a}\Delta a + \frac{\partial\lambda_s}{\partial i}\Delta i \\ \frac{\partial\lambda_s}{\partial a} &= R_{\oplus}(\omega_{\oplus} - \dot{\Omega})\frac{\partial\rho_{\Omega}}{\partial a} - \frac{\partial\dot{\Omega}}{\partial a}R_{\oplus}\rho_{\Omega} \\ \frac{\partial\lambda_s}{\partial i} &= R_{\oplus}(\omega_{\oplus} - \dot{\Omega})\frac{\partial\rho_{\Omega}}{\partial i} - \frac{\partial\dot{\Omega}}{\partial i}R_{\oplus}\rho_{\Omega} \end{aligned} \quad (11-24)$$