Paper AAS 07-127

Validation of SGP4 and IS-GPS-200D Against GPS Precision Ephemerides

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17th AAS/AIAA Space Flight Mechanics Conference

Sedona, Arizona **January 28–February 1, 2007**

AAS Publications Office, P.O. Box 28130, San Diego, CA 92198

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VALIDATION OF SGP4 AND IS-GPS-200D AGAINST GPS PRECISION EPHEMERIDES

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Many applications use the NORAD SGP4 orbital model for predicting satellite ephemerides and often these applications require knowledge of the errors associated with those predictions. Unfortunately, the SGP4 orbital data, in the form of two-line element (TLE) sets, does not provide any kind of accuracy information. Some approaches have been published which purport to estimate these errors by performing consistency or abutment checks, but they do not validate their assumptions or provide any validation by comparison to high-accuracy ephemerides. This paper will assess the suitability of these approaches by comparing SGP4 ephemerides to precision ephemerides available for the GPS constellation.

INTRODUCTION

As the most complete source of orbital element information available to the public on the full catalog of objects in Earth orbit today, the NORAD two-line element sets (TLEs)—together with the associated SGP4 orbital model—are used in a wide variety of orbit propagation tasks. As with any type of prediction, however, analysts need to be able to estimate the uncertainty associated with these predictions in order to quantify the level of confidence in the resulting analysis. Applications such as conjunction analysis, in particular, are very dependent on being able to specify the prediction uncertainty in order to facilitate critical decision making.

Unfortunately, TLEs do not come with an associated covariance estimate. That means users are faced with having to either ignore the inherent uncertainty associated with the SGP4 predictions or to develop other methods for estimating the covariance a posteriori.

Several papers have addressed how this might be done in the past, but each of these approaches has shortcomings which prevent their implementation in practical applications. In the MAESTRO approach,[1] the accuracy of the TLE data is compared to the observation data used to generate the TLE data in the first place. This approach presents a variety of problems, since not only is the calibration source not independent of the data,

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but the data used is not available to the majority of researchers. As such, it does nothing to address the fundamental problem.

In the COVGEN approach,[2] TLEs are compared to each other to determine how TLE predictions change with the propagation interval. While this approach is suitable for use by the general public, since it uses only publicly available TLE data, it makes several assumptions regarding the properties of the errors which are not properly validated. In particular, assumptions are made that the TLE prediction errors are unbiased and that error growth is independent of whether the TLEs are propagated forward or backward in time relative to the TLE epoch. As we shall see soon, these assumptions are not supported by a closer examination of the data.

OBJECTIVES

The goal of this paper is to examine the general approach proposed in COVGEN to assess its general suitability for developing covariance estimates for applications which require them. To accomplish this goal, we will use an independent truth data set of much higher accuracy than that provided by SGP4 with TLE data and analyze the SGP4 prediction error as a function of propagation interval. We will then generate TLE consistency results, as done in COVGEN, to see how these results compare to our truth results. We will also examine whether SEM or Yuma almanac data for the GPS constellation provide any additional capability for this particular set of satellites.

Through this process, several characteristics of the TLE consistency data should become clearer, suggesting ways to modify or improve on the COVGEN approach for estimating TLE covariance data.

TEST DATA

In order to properly test the accuracy of the GPS almanac and TLE data, it is necessary to choose an appropriate data set which mitigates spurious results which might occur from maneuvers or other anomalies of any of the operational GPS satellites. As such, the operational status of the GPS constellation was assessed for 2006 by examining the NANUs (Notice Advisory to NAVSTAR Users) issued by the GPS Operations Center.

For this study, each operational satellite is referred to by its PRN (Pseudo-Random Noise) Number, which can range from 1-32. As of the end of 2006, PRN 32 had not yet been assigned to an operational satellite. PRNs 12 and 31 were not launched until late 2006 and are not further considered for this study. PRNs 3, 5, 6, 15, 25, 29, and 30 were discarded because they experienced 10 or more outage days during 2006, indicating the potential for problems which might affect this analysis. Figure 1 summarizes these results by showing outages for each satellite in red, with those satellites not used for this study in yellow.

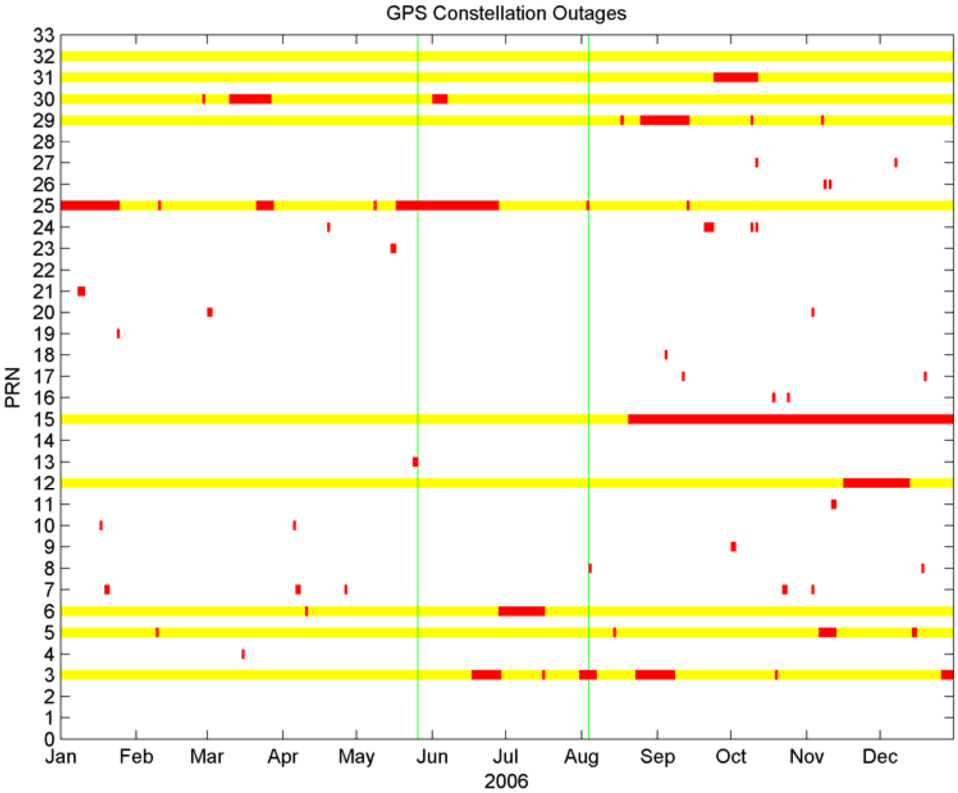

Figure 1. GPS Constellation Outages for 2006

Of the remaining 22 operational GPS satellites, the period from Day 147 to Day 217 (between the green vertical lines in Figure 1) was the longest period where none of these satellites experienced an outage. As a result, it was decided to select the period from Day 150 (May 30) to the end of Day 210 (Jul 29) for this analysis. SEM almanacs for GPS Week 353, TOA 233472 to GPS Week 361, TOA 589824 were used for the almanac data and TLEs from Day 150 to Day 210 were selected for all 22 satellites.

TRUTH DATA

In order to have confidence in our analysis of various estimation techniques, it is vital to have a reliable source of truth data with which to work. Because of its use for a variety of high-precision navigation and timing applications, the operational GPS constellation is well suited for providing this type of data. The operational GPS constellation is closely monitored on a continuous basis by a global network of sensors, including laser ranging. These observations are then post-processed to provide high-precision ephemerides which are used in the most demanding navigation and timing applications. These data are provided on a regular basis and readily available to the public.

The US National Geospatial-Intelligence Agency (NGA) makes GPS Satellite Precise Ephemeris (PE) data available daily, in the standard SP3 format, via the GPS Division of the GeoInt Sciences Office.[†] The data is provided as Earth-centered Earth-fixed (ECEF) trajectories using the WGS 84 coordinate system, providing position and velocity at 15-minute intervals, referenced to GPS time. The advertised accuracy of these data can be found on the NGA/IGS GPS Orbit (Ephemeris) Comparison page.[‡]

For the data used in this study, the mean difference is 16.8 cm with a standard deviation of 1.1 cm. Combined with the advertised accuracy of the IGS (International GNSS Service) products, which are calibrated with satellite laser ranging measurements, of less than 5 cm,[§] the overall accuracy should be well within 25 cm. This error is well below the level of the TLE error, as will be seen later.

METHODOLOGY

The first step in the analysis was to take the precise ephemeris data for the entire period and generate individual data files by PRN. This step was pretty straightforward, since the SP3 data already provides ECEF position at 15-minute (GPS Time) intervals for all operational GPS satellites. It was only necessary to separate out the data by satellite and format it for later use. No conversions were applied to the position or time during this process.

The next step was to take the SEM almanac data and generate ephemerides for each almanac for the entire analysis period. In order to best match the precise ephemerides, this data was produced at the same 15-minute GPS Time points in ECEF position. Generation was in accordance with IS-GPS-200D.[3]

It should be noted that similar data, the Yuma almanacs, could have been used, but analysis in this study showed that predictions using this data in accordance with IS-GPS-200D yielded results which agreed with those from the SEM almanacs to better than 1 mm—far better than the accuracy of even the precise ephemerides. As such, only the SEM almanac data set was used for this analysis to avoid redundancy.

The data from each data set were generated as STK external data sets, so that STK could be used to perform the necessary coordinate transformations to allow comparison of the almanac ephemerides to the precise reference trajectory in the RIC (radial, in-track, cross-track) coordinate system of the reference trajectory. The RIC position of each data point from each almanac ephemeris was then collected as a function of propagation interval (forward or backward) from the reference time (TOA) for that almanac.

Next, the data for each TLE was propagated, using NORAD SGP4 orbital model now implemented in STK.[4] STK performed the appropriate coordinate conversions to calculate the RIC position of each TLE ephemeris point, on the same 15-minute GPS Time points as used in the precise ephemerides. As with the almanac data, the RIC

[†] http://earth-info.nga.mil/GandG/sathtml/PEexe.html.
[‡] http://earth-info.nga.mil/GandG/sathtml/ngaigscompare.html.
[§] http://igscb.jpl.nasa.gov/components/prods.html.

position of each point was then collected as a function of the propagation interval from the epoch of the TLE.

RESULTS: ALMANAC COMPARISON TO PRECISE EPHEMERIDES

For each of the 22 satellites selected for this study, 60 SEM almanacs were used with IS-GPS-200D to generate ephemeris files for the period 2006 May 30 00:00:00 GPST to 2006 Jul 30 00:00:00 GPST at 15-minute intervals (a total of 1,320 files). RIC positions for each satellite, referenced to the corresponding precise ephemeris were calculated for a period of ±15 days from the TOA of each almanac. The RIC data was then plotted as a function of propagation interval. Page restrictions prohibit inclusion of all the resulting plots, so two representative plots (PRN10 and PRN11) are chosen as examples. All of the plots, however, are available for review at http://celestrak.com/publications/AAS/07-127/.

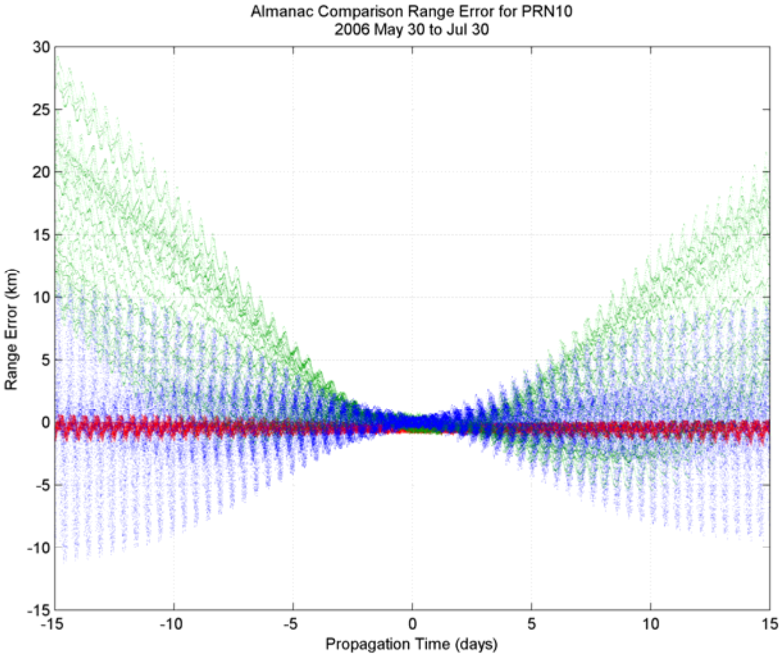

Figure 2. PRN10 Almanac Comparison to Precise Ephemeris

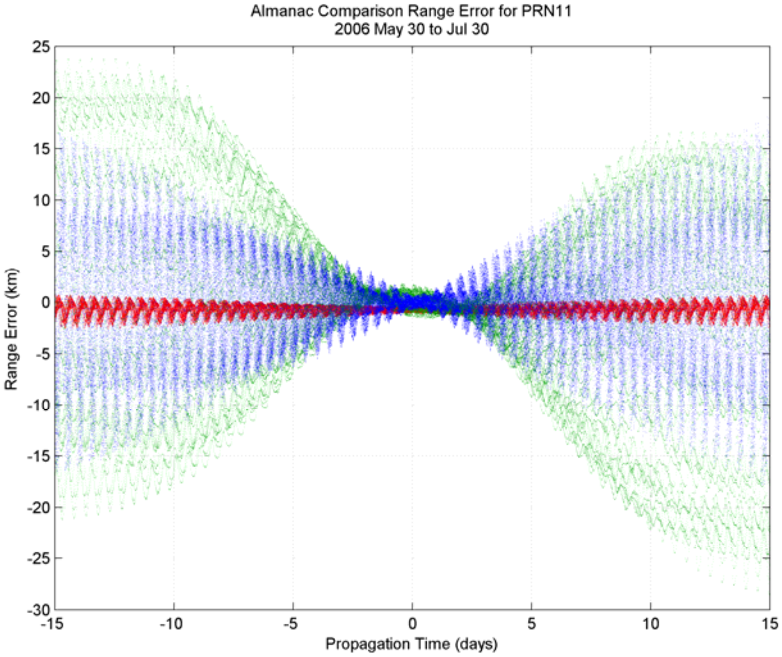

Figure 3. PRN11 Almanac Comparison to Precise Ephemeris

All plots show radial errors in red, in-track errors in green, and cross-track errors in blue. Periodic oscillations seen in the errors are a result of the mismatch between the simplistic propagation model used for the almanacs (and, as we will see shortly, for TLEs with SGP4) compared to the full geopotential modeling in the precise ephemerides.

Simplified views of Figure 2 and Figure 3 are provided in Figure 4 and Figure 5, respectively, showing only the fundamental statistical results, binned to the closest day in propagation time. Each plot shows the mean value for each bin along with a one-sigma error bar for that bin. These simplified plots should make it easier to discern the bias and variance characteristics of each plot.

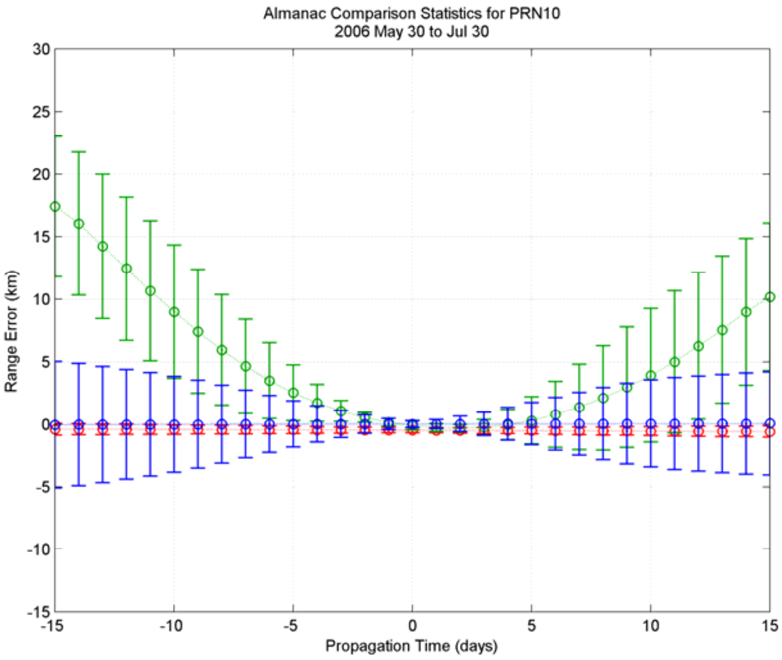

Figure 4. PRN10 Almanac Comparison Statistics

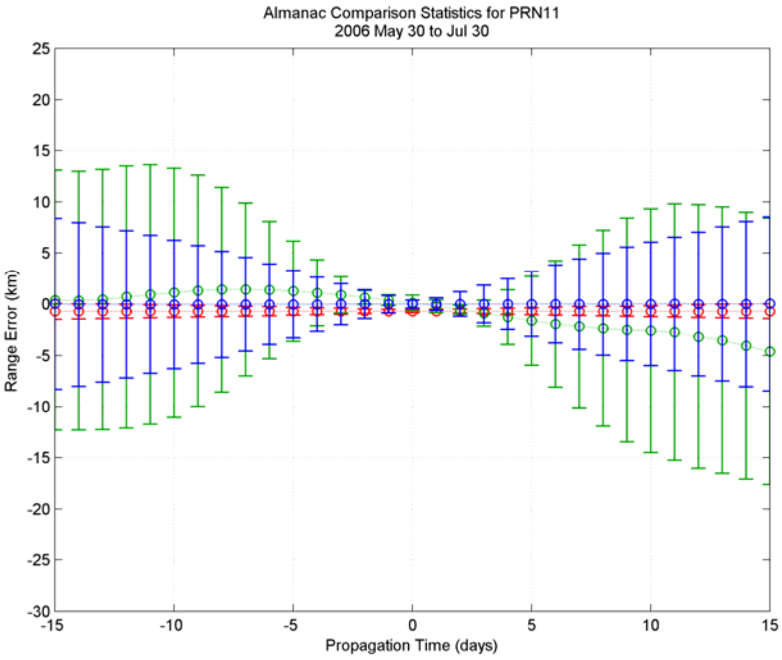

Figure 5. PRN11 Almanac Comparison Statistics

From a review of all plots, the following patterns were discerned:

1. The in-track error was generally dominant, followed by the cross-track error. The radial error was typically far smaller than either of the other errors.

2. The radial and cross-track errors were not significantly biased, with the range of errors typically including the reference trajectory at the one-sigma level. The in-track error, however, showed some slight bias in most cases (Figure 3 and Figure 5) and significant bias in others (Figure 2 and Figure 4).

3. Errors were generally symmetric with respect to propagation direction.

4. Errors grow as a function of propagation interval (both forward and backward).

RESULTS: TLE COMPARISON TO PRECISE EPHEMERIDES

For each of the 22 satellites, all TLEs for Days 150 to 210 were used with SGP4 to compare to the precise ephemerides (the number of TLEs used for each satellite is shown in Table 1) for the period 2006 May 30 00:00:00 GPST to 2006 Jul 30 00:00:00 GPST at 15-minute intervals. RIC positions for each satellite, referenced to the corresponding precise ephemeris, were calculated for a period of ±15 days from the epoch of each TLE. As before, the RIC data was then plotted as a function of propagation interval. Representative plots are shown in Figure 6 through Figure 9 below.

Table 1. PRN Mapping

PRN	International Designator	NORAD Catalog Number	TLEs
01	1992-079A	22231	80
02	2004-045A	28474	95
04	1993-068A	22877	82
07	1993-032A	22657	93
08	1997-067A	25030	78
09	1993-042A	22700	83
10	1996-041A	23953	88
11	1999-055A	25933	84
13	1997-035A	24876	86
14	2000-071A	26605	81
16	2003-005A	27663	84
17	2005-038A	28874	83
18	2001-004A	26690	81
19	2004-009A	28190	81
20	2000-025A	26360	89
21	2003-010A	27704	83
22	2003-058A	28129	86
23	2004-023A	28361	84
24	1991-047A	21552	86
26	1992-039A	22014	149
27	1992-058A	22108	81
28	2000-040A	26407	82

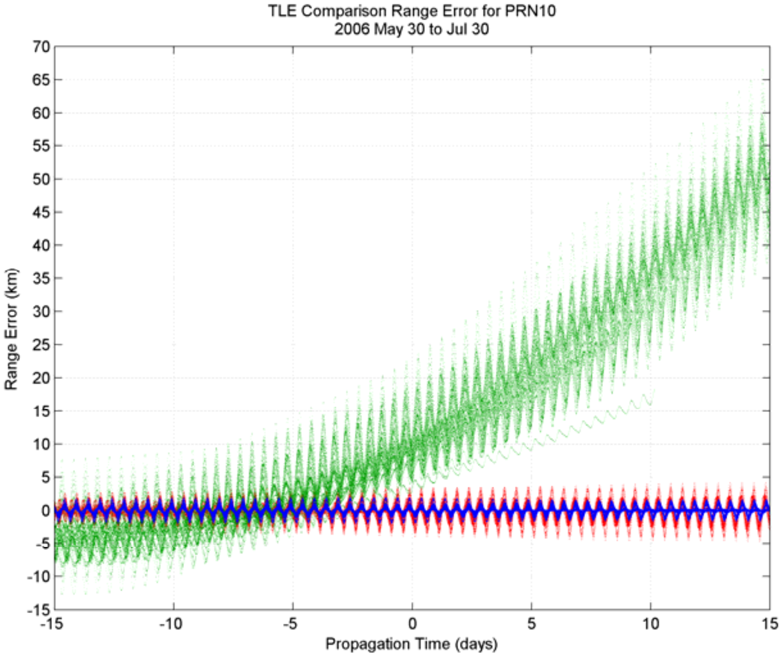

Figure 6. PRN10 TLE Comparison to Precise Ephemeris

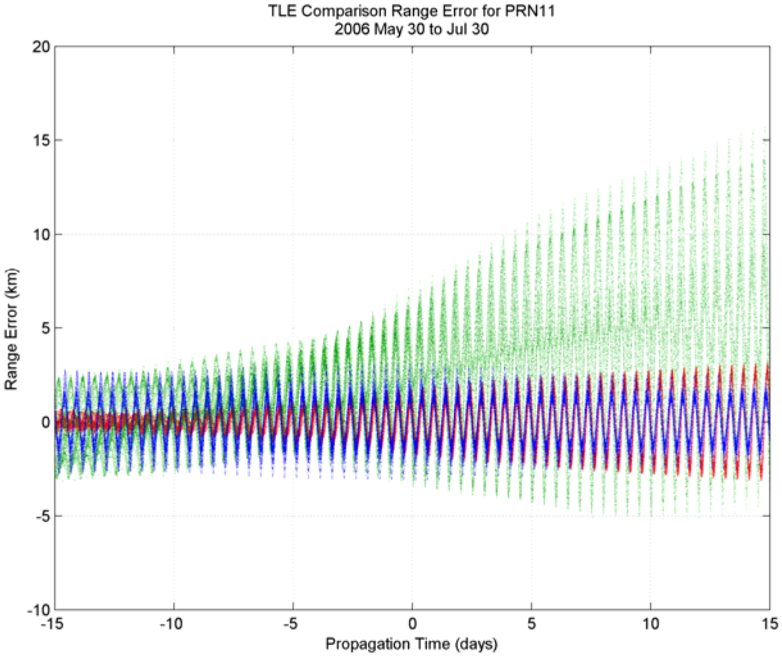

Figure 7. PRN11 TLE Comparison to Precise Ephemeris

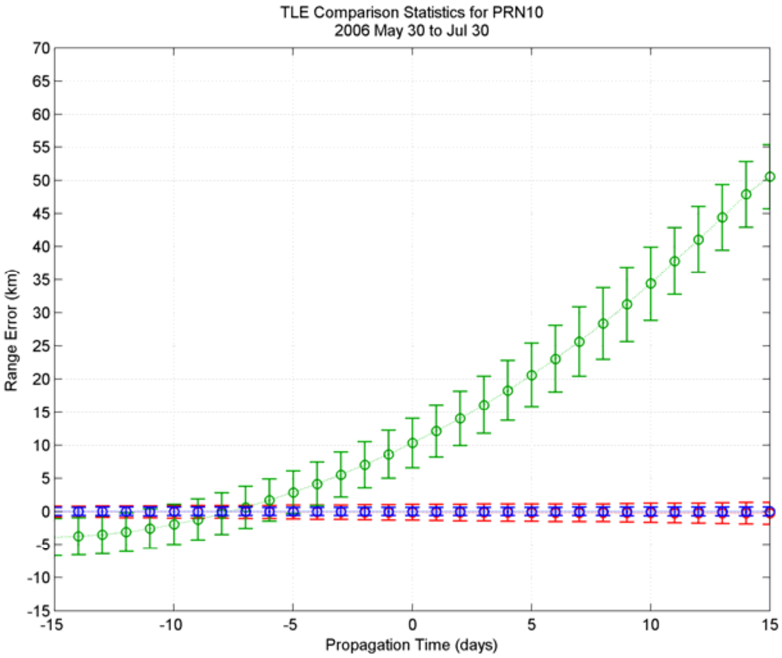

Figure 8. PRN10 TLE Comparison Statistics

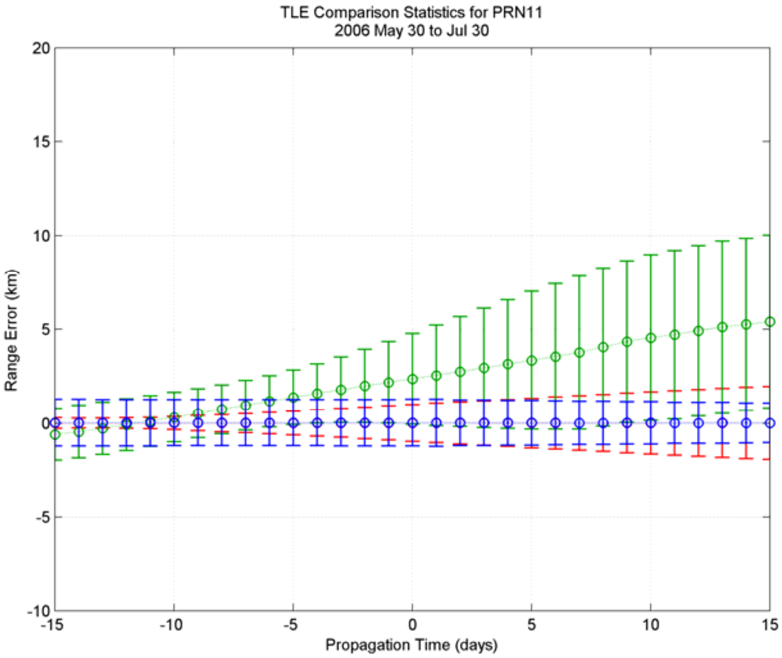

Figure 9. PRN11 TLE Comparison Statistics

From a review of all TLE comparison plots, the following patterns were discerned:

1. The in-track error was again generally dominant, although in this case it was typically followed by radial error and then cross-track error.

2. There were significant biases in the in-track error. There were some slight biases in the other two coordinates, in some cases.

3. Errors were clearly not symmetric with respect to propagation direction.

4. While the bias grew with time, often the variance did not.

Many of these effects were masked in the COVGEN report, since the errors were assumed to be symmetric in both time and direction.

RESULTS: ALMANAC AND TLE COMPARISONS

The error profiles of the almanac and TLE propagators show some significant differences. While the maximum error over the ±15 day interval is comparable, the distribution of the error with time is considerably different. In addition, the minimum error for the TLEs does not seem to occur at 0 propagation time, as is the case with the almanacs, nor is the minimum one-sigma error as small for the TLEs.

RESULTS: TLE CONSISTENCY

A TLE consistency analysis was performed for all satellites to attempt to determine whether such an analysis could reasonably approximate the error distribution seen in the comparison of the TLE predictions to the precise ephemerides. For each satellite, each pair of TLEs was used to calculate the TLE positions at the epoch times of the two TLEs.

For TLE_i and TLE_j, with epoch times t_i and t_j, TLE_i and TLE_j are each propagated to t_i and t_j. For TLE_i, propagating to t_i is a propagation time of 0 and propagating to t_j is a propagation time of $t_j - t_i$. For TLE_j, propagating to t_i is a propagation time of $t_i - t_j$ and propagating to t_j is a propagation time of 0. The position for TLE_i is assumed to be 'true' at t_i and the position of TLE_j is assumed to be 'true' at t_j. These reference states are then used to calculate the RIC position of the other state.

Following this process for a set of n TLEs, there are

$$\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$$

possible combinations of TLEs. This number was further reduced by only using pairs where $|t_i - t_j| \leq$ 15 days.

The resulting plots are shown in Figure 10 through Figure 13 below. While the high-frequency periodic effects are lost as a result of the low-frequency of TLE updates and there is an artificial pinching of the error around 0 propagation time, as a result of our assumptions, a comparison of the TLE consistency plots to the TLE comparison plots shows a good match of the overall characteristics.

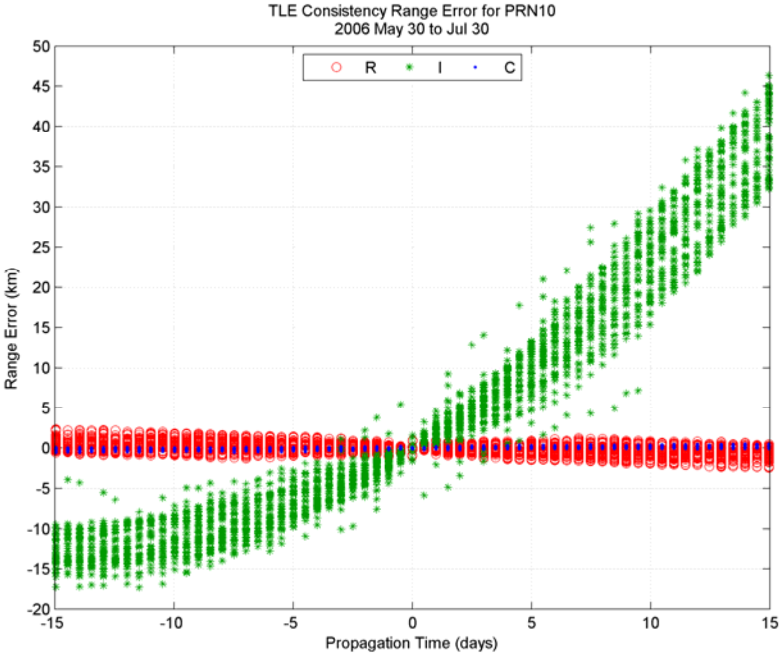

Figure 10. PRN10 TLE Consistency Plot

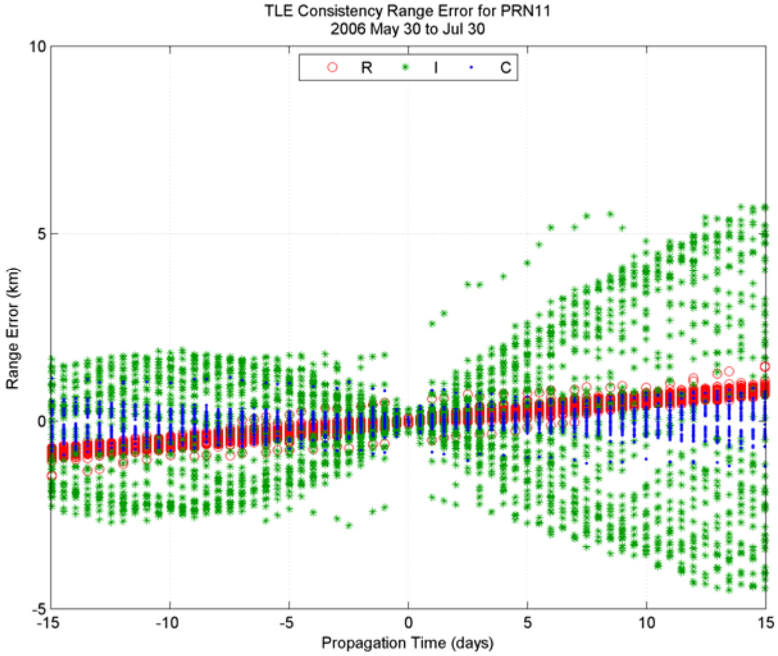

Figure 11. PRN11 TLE Consistency Plot

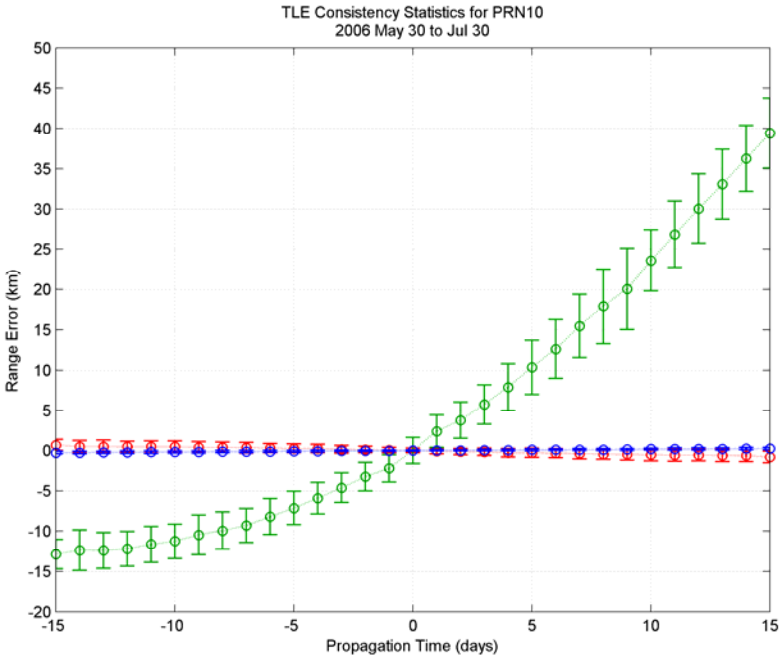

Figure 12. PRN10 TLE Consistency Statistics

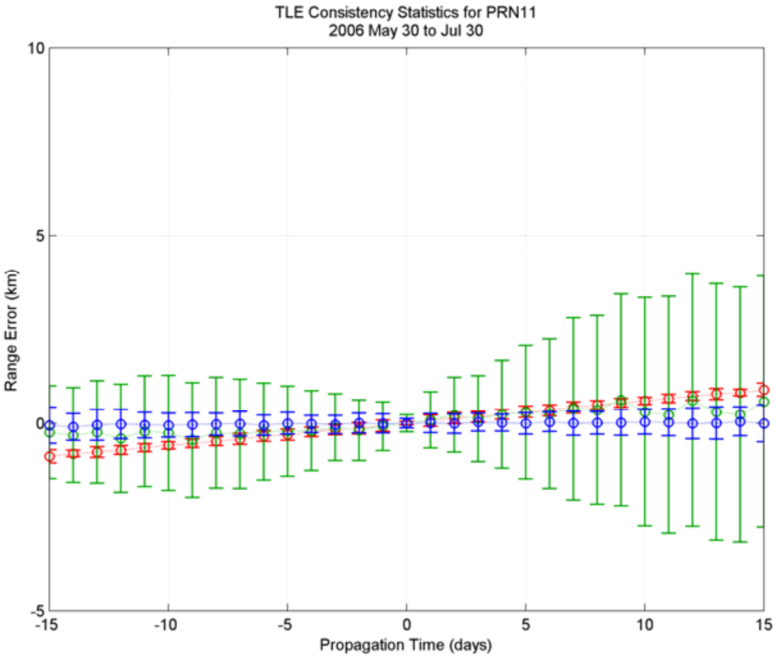

Figure 13. PRN11 TLE Consistency Statistics

CONCLUSIONS

As a result of this analysis, we can conclude:

1. Errors associated with almanac and TLE predictions are comparable, at least within 15 days of the epoch, although almanac predictions are much better near the epoch.

2. TLE consistency analysis does reasonably approximate the true error of a TLE prediction, both in propagation time, direction, and overall magnitude, although it does underestimate it near the epoch.

3. There are clear biases in the TLE errors which, if not accounted for, can lead to an overestimation of the error. It should, however, be possible to improve a TLE estimate by estimating and removing this bias. Not only would the estimate improve but the associated error would decrease, thereby increasing the overall confidence in the resulting prediction.

4. Error characteristics for satellites in similar orbits can be considerably different. As such, the error characteristics of each satellite should be determined independently.

FUTURE RESEARCH

As a result of these conclusions, a follow-on effort is planned to attempt to improve TLE accuracy along with providing an estimate of the covariance. While it would be possible using the TLE consistency approach to simply estimate the bias and remove it, along with estimating the covariance, such an approach would require any current processes which use TLEs for predictions to be modified to incorporate a time-varying bias. Rather than force all such processes to be modified, this follow-on effort plans to attempt to remove the bias and estimate the covariance using a Kalman filter and then generate a refined TLE which could be used for analysis without modifying existing processes.

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